Faktoriál – rovnice a nerovnice

Příklady

① Řešte v $\mathbb{N}_0$

a) $(x+1)! = 110(x-1)!$

Nutné podmínky: $n!$ def. pro $n \in \mathbb{N}_0$ $(n \geq 0 \wedge \mathbb{N}_0)$

$$(x+1)\cdot x\cdot(x-1)! = 110(x-1)! \quad \Big|: \underbrace{(x-1)!}_{\text{lze, } (x-1)! \neq 0 \text{ pro každé } x\in D}$$

$$\left[\begin{matrix} x+1\geq 0 & \wedge & x-1\geq 0 & \wedge & x\in\mathbb{N}_0 \\ x\geq -1 & & x\geq 1 & & \end{matrix}\right]$$

$$x^2+x = 110$$

$$x^2+x-110 = 0$$

$$D = \mathbb{N}$$

$$(x+11)(x-10) = 0$$

$$x_1 = -11_{\notin D} \quad x_2 = 10_{\in D} \qquad K = \{10\}$$

b) $(n-1)!\cdot n! = (n-2)!\cdot(n-1)^2\cdot n$

$$\left[\underbrace{\begin{matrix} n-1\geq 0 & \wedge & n-2\geq 0 & \wedge & n\in\mathbb{N}_0 \\ n\geq 1 & & n\geq 2 & & \end{matrix}}_{\mathbb{N}_0-\{0,1\}=\mathbb{N}-\{1\}}\right]$$

$$(n-1)\cdot(n-2)!\cdot n! = (n-2)!\cdot(n-1)^2\cdot n$$

$$n! = (n-1)\cdot n$$

$$n\cdot(n-1)\cdot(n-2)! = (n-1)\cdot n \quad \Big|: \underbrace{n\cdot(n-1)}_{\text{lze, } n\neq 0,\ n\neq 1 \text{ pro každé } n\in D}$$

$$(n-2)! = 1$$

$$D = \mathbb{N}-\{1\}$$

$$n-2 = 0 \qquad n-2 = 1$$

$$n_1 = 2_{\in D} \qquad n_2 = 3_{\in D} \qquad K = \{2,3\}$$

[obecně $n! = 1 \Leftrightarrow n = 0 \vee n = 1$
$0! = 1 \vee 1! = 1$]

c) $\frac{x\cdot(x+3)!}{(x+2)!} + x^2 = 14$

$$\left[\underbrace{\begin{matrix} x+3\geq 0 & x+2\geq 0 & x\in\mathbb{N}_0 \\ x\geq -3 & x\geq -2 & \end{matrix}}_{x\in\mathbb{N}_0}\right]$$

$$\frac{x\cdot(x+3)\cdot(x+2)!}{(x+2)!} + x^2 = 14$$

$$x(x+3)+x^2 = 14$$

$$x^2+3x+x^2 = 14$$

$$D = \mathbb{N}_0$$

$$2x^2+3x-14 = 0$$

$$(a = 2, b = 3, c = -14)$$

$$x_{1,2} = \frac{-b\pm\sqrt{b^2-4ac}}{2a} = \frac{-3\pm\sqrt{9+4\cdot 2\cdot 14}}{4} = \frac{-3\pm\sqrt{121}}{4} = \frac{-3\pm 11}{4} = \begin{cases} \frac{8}{4} = 2_{\in D} \\ -\frac{14}{4}_{\notin D} \end{cases}$$

$$K = \{2\}$$

d) $(n+1)! = 42\cdot(n-1)!$

$$\left[\underbrace{\begin{matrix} n+1\geq 0 & n-1\geq 0 & n\in\mathbb{N}_0 \\ n\geq -1 & n\geq 1 & \end{matrix}}_{n\in\mathbb{N}_0-\{0\}=\mathbb{N}}\right]$$

$$(n+1)\cdot n\cdot(n-1)! = 42\cdot(n-1)! \quad \Big|: \underbrace{(n-1)!}_{\neq 0}$$

$$n^2+n = 42$$

$$n^2+n-42 = 0$$

$$D = \mathbb{N}$$

$$(n+7)\cdot(n-6) = 0$$

$$n+7 = 0 \qquad n+6 = 0$$

$$n_1 = -7_{\notin D} \qquad n_2 = 6_{\in D} \qquad K = \{6\}$$

② Řešte v $\mathbb{R}$

$\left[\begin{matrix} \textit{exponenciální rovnice} \\ a^x = a^y \Leftrightarrow x = y \end{matrix}\right]$

a) $(5!)^x = \frac{(4!)^x}{25}$ $\quad$ D = R

$\left[\begin{matrix} (a \cdot b)^r = a^r \cdot b^r & \underbrace{\frac{1}{a^r} = a^{-r}}_{\text{pro } a \neq 0} \\ a^r \cdot a^s = a^{r+s} & \underbrace{a^0 = 1}_{\text{pro } a \neq 0} \end{matrix}\right]$

1. zp.

$25 \cdot (5 \cdot 4!)^x = (4!)^x$

$5^2 \cdot 5^x \cdot (4!)^x = (4!)^x$

$5^{2+x} = 1$

$5^{2+x} = 5^0$

$2 + x = 0$

$x = -2$

2. zp.

$(5 \cdot 4!)^x = (4!)^x \cdot 5^{-2}$

$5^x \cdot (4!)^x = (4!)^x \cdot 5^{-2}$

$5^x = 5^{-2}$

$x = -2$

Zkouška (pro ověření)

$L = (5!)^{-2} = \frac{1}{(5!)^2}$ $\quad$ $P = \frac{(4!)^{-2}}{25} = \frac{1}{5 \cdot 5 \cdot (4!)^2} = \frac{1}{5 \cdot 4! \cdot 5 \cdot 4!} = \frac{1}{(5!) \cdot (5!)} = \frac{1}{(5!)^2}$ $\quad$ L = P

$K = \{-2\}$

b) $(5!)^{x+1} = (6!)^{x-1} \cdot 20^2$ $\quad$ D = R

1. zp.

$(5!)^x \cdot (5!)^1 = (6!)^x \cdot (6!)^{-1} \cdot 20^2$

$(5!)^x \cdot 5! = \frac{(6!)^x}{6!} \cdot 20^2$

$5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 6^x \cdot \frac{20 \cdot 20}{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}$

$3 \cdot 2 \cdot 1 = 6^x \cdot \frac{1 \cdot 1}{6 \cdot 3 \cdot 2 \cdot 1}$

$6^3 = 6^x$

$x = 3_{\in D}$ $\quad$ $K = \{3\}$

2. zp.

$(5!)^x \cdot 5! = \frac{(6 \cdot 5!)^x}{6!} \cdot 20^2$

$(5!)^x \cdot 5! = \frac{6^x \cdot (5!)^x}{6!} \cdot 20^2$ (dále viz 1. zp.)

③ Řešte v $\mathbb{N}_0$

a) $n! \cdot (n+3)! > (n+1)! \cdot (n+2)!$

$\left[\underbrace{\begin{matrix} n+3 \geq 0 & \wedge\ n+1 \geq 0 & \wedge\ n+2 \geq 0 & \wedge\ n \in \mathbb{N}_0 \\ n \geq -3 & n \geq -1 & n \geq -2 & \end{matrix}}_{n \in \mathbb{N}_0}\right]$

$n! \cdot (n+3) \cdot (n+2)! > (n+1) \cdot n! \cdot (n+2)!$ $\quad \Big|: \underbrace{n! \cdot (n+2)!}$ $\quad$ $D = \mathbb{N}_0$

lze, výraz $\neq 0$ pro každé $n \in D$;
$n! > 0$, $(n+2)! > 0$, neměníme znak nerovnosti

$n + 3 > n + 1$

$3 > 1$

$(0n > -2$ pl.$)$ $\quad$ $K = D = \mathbb{N}_0$

b) $n \cdot (n-2)! > (n-1)!$

$\left[\begin{matrix} n-2 \geq 0 & \wedge\ n-1 \geq 0 & \wedge\ n \in \mathbb{N}_0 \\ n \geq 2 & \wedge \quad n \geq 1 & \end{matrix}\right]$

$n \cdot (n-2)! > (n-1) \cdot (n-2)!$ $\quad \Big|: \underbrace{(n-2)!}$ $\quad$ $D = \mathbb{N} - \{1\}$

lze, výraz $\neq 0$ pro každé $n \in D$;
$(n-2)! > 0$, neměníme znak nerovnosti

$n > n - 1$

$0n > -1$ pl. $\quad$ $K = D = \mathbb{N} - \{1\}$

c) $12n! + 3n \cdot n! \geq (n+2)!$ $\quad$ $[n \geq 0,\ n+2 \geq 0, n \in \mathbb{N}_0]$

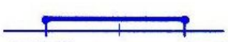
$n! \cdot (12 + 3n) \geq (n+2) \cdot (n+1) \cdot n!$ $\quad \Big|: \underbrace{n!}$ $\quad$ $D = \mathbb{N}_0$

lze, výraz $\neq 0$ pro každé $n \in D$;
$n! > 0$ neměníme znak nerovnosti

$12 + 3n \geq n^2 + 3n + 2$

$12 \geq n^2 + 2$

$n^2 \leq 10$



$|n| \leq \sqrt{10}$ $\quad$ $\sqrt{-10}$ $\quad$ 0 $\quad$ $\sqrt{10}$ $\quad$ ale v $\mathbb{N}_0$ $\quad$ $\Rightarrow K = \{0, 1, 2, 3\}$